Layering and Staircase Formation in Fluid Dynamics

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One of the most interesting features of certain turbulent fluid dynamical systems is their tendency to form layers or "staircases", in which a key physical quantity, such as the density, exhibits a staircase structure with depth, rather than being more smoothly distributed as one might expect in a mixing flow. This layering phenomenon occurs in a variety of systems that appear to be rather different physically; a research question of considerable interest therefore is whether, at heart, the underlying physics of layering is the same in these different systems. Understanding staircase formation is not only of intrinsic scientific interest but is vital to understanding turbulent transport. Layered and unlayered systems have very different transport properties; thus it is essential to understand how and when layering can occur in order that turbulent transport can be realistically parameterised in large oceanographic or atmospheric models.

In the context of atmospheres and oceans there are two particular systems of interest that are known to be susceptible to layering. One is double-diffusive convection – in which the buoyancy depends on two different components that diffuse at different rates. in the oceans, buoyancy depends on both heat and salt, with the diffusion of heat being much more effective than that of salt; the process is then known as *thermohaline* or *thermosolutal* convection (for a discussion of all aspects of double diffusive convection, see the recent monograph by Radko 2013). In this system, convection can be driven either by an unstable solutal gradient with a stable thermal gradient (which occurs in warmer oceans and is known as the *salt fingering* regime) or, alternatively, by an unstable thermal gradient with a stable solutal gradient (the situation in colder oceans, known as the *diffusive* regime). In both these cases, in certain parameter regimes, the density (and also the temperature and salinity), rather than varying gradually with depth, is observed to take on a staircase structure, with layers of roughly uniform density separated by quite pronounced interfaces; these layers appear to be remarkably resilient, being able to survive disturbance by oceanographic waves. Figure 1 shows the results of a numerical simulation of turbulent thermohaline convection in the diffusive regime; layers (shown here in salinity) are clearly visible. Interestingly, a similar process is believed to be of importance in stellar cores, where the competing elements are thermal and compositional gradients.

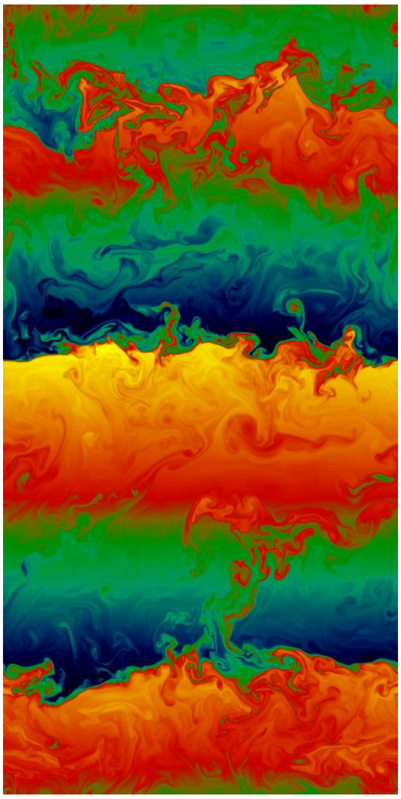
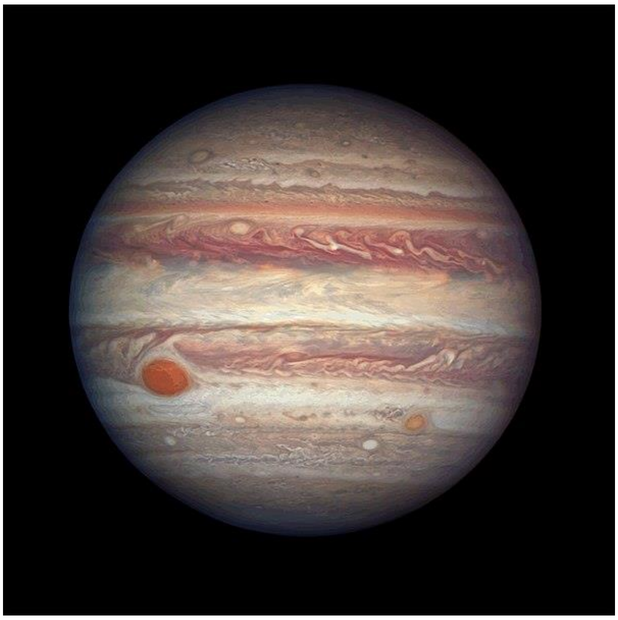


Figure 1: Salinity intensity, calculated from a numerical simulation of thermohaline convection in the diffusive regime (from Hughes & Brummell 2019). Note that although the flow is turbulent, distinct layers of salinity have formed.

The other layering process of great interest in geophysics takes place in the atmosphere, when the influence of rotation and stratification are strong, and concerns the formation of layers (staircases) in *potential vorticity*; the manifestation of this staircase is the appearance of strong jets. This process is relevant not only for our own atmosphere, but in many other stratified systems; possibly the most striking is in the outer layers of Jupiter's atmosphere, which is characterised by a strongly banded structure (see Figure 2).



Jets in Jupiter's outer atmosphere; this is a manifestation of a staircase in potential vorticity.

Over the past few years, considerable progress has been made in our understanding of layering in double diffusive convection (e.g. Stellmach et al 2011) and rotating stratified convection (e.g. Dritschel & McIntyre 2008). That said, there is still considerable debate over the physics of layering, and little consensus over common ground between the various different physical systems. This project will explore, through simplified models – following Balmforth et al 1998 – the entire nature of staircase formation: namely, the physical ingredients necessary for their formation, the scale at which they initially form, and how subsequent staircase mergers occur. The model equations will, initially at least, be nonlinear partial differential equations in one spatial direction (height, for example, for the case of thermohaline convection) and time. These will be simpler than the full equations of three-dimensional fluid dynamics, but will nonetheless allow a detailed analysis and understanding of what exactly is needed to provide layering. The project will involve a combination of analytical and asymptotic approaches, together with numerical solutions of the model equations.

Objectives

The ultimate aim of this project is to understand the fundamental mechanisms underlying the formation of layers (staircases) in various turbulent fluid dynamical systems. In particular, the project will involve:

1. Deriving simplified model equations (PDES in one spatial direction and time) to describe double-diffusive convection and stratified rotating turbulence.
2. Analysing the solutions of these equations through a combination of computational and asymptotic techniques, the latter utilising the disparity in timescales that can be present.
3. Exploring, through the model equations, common features between systems that, ostensibly appear rather different.
4. Analysing the influence of layers on the turbulent transport of crucial physical quantities (e.g. heat and salt in thermohaline convection).
5. Extending the results to other systems that are known to exhibit staircases – such as the so-called “ExB” staircases that are conjectured to exist in the plasma of magnetically confined fusion devices.

Potential for high impact outcome

Obtaining a full understanding of the process of layer (staircase) formation in turbulent fluid dynamical systems remains a great challenge. Although much is known about the systems separately, either by observation, experiments or numerical simulations, a full theoretical understanding remains elusive. Thus making progress in this area is potentially of very high impact, with implications not only for oceanographic and atmospheric dynamics, but also in astrophysics (mixing in stellar cores and jet formation in planetary atmospheres) and plasma fusion physics (with so-called E x B staircases observed in tokamaks). We therefore anticipate the project generating several papers being suitable for submission to a high impact journal.

Training

The student will work under the supervision of David Hughes and Sandro Azaele within the School of Mathematics. The student will thus be a member of two very strong research groups within the School: the astrophysical and geophysical fluid dynamics group and the nonlinear dynamics group.

Both groups have weekly seminars, in addition to the Applied Mathematics seminars, and the AGFD group also holds a weekly discussion meeting in which scientific papers of interest are discussed. A wide variety of courses are on offer to PhD students, provided both by the School of Maths and the faculty (http://www.emeskillstraining.leeds.ac.uk/). These range from level 5 (Masters level) courses, to specialised computational training, through to training in more general aspects of the PhD, such as managing your degree and preparing for your viva. Through the project, the student will be trained not only to tackle the particular problem at hand, but will also acquire valuable analytical and computational skills.

Student profile

The student should have a strong background in either applied maths or physics, certainly with some knowledge of, and interest in, fluid dynamics. The project will involve the study of nonlinear partial differential equations, both analytically, using asymptotic techniques, and computationally. Some background in mathematical methods, particularly asymptotic methods, and PDEs would therefore be helpful.

References

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- ***Radko**, T. (2013) Double-Diffusive Convection, Cambridge University Press.*
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